# Rare Shocks vs. Non-linearities: What Drives Extreme Events in the Economy? Some Empirical Evidence

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# The aim of the paper

- To examine roles of extreme shocks and non-linearities during extreme events in the economy.
- Extreme/rare shocks:
  - Does occurrence of large shocks corresponds to normal distribution?
  - t-distributed shocks (fat tails)
- Non-linearities:
  - regime switch in shock propagation mechanism and shock volatility
- Extreme events:
  - crises, downturns, crashes
  - focus on dynamics of such events
- Need to take into account credit/financial markets (and real economy).

# Motivation

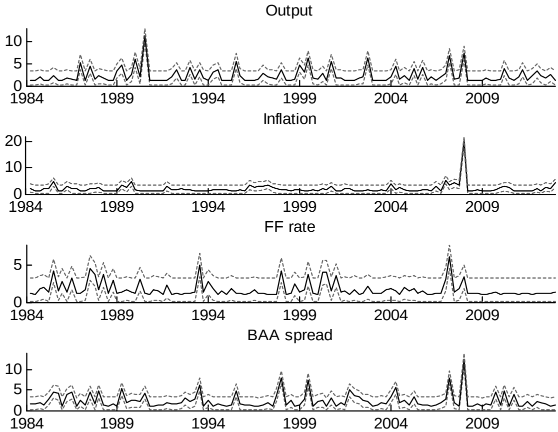



Figure: Reduced form residuals (absolute value) in standard deviation units.

# Motivation

- Model used in estimation of residuals: linear, small-scale, Gaussian, with constant shock volatility.
- Each characteristic can represent a wrong assumption.
- Need to account for all types of non-linearity and fat tails simultaneously - ignoring one can falsely suggest presence of the other.
- Important for policy makers:
  - shocks are unexpected - cannot be dealt with by some preemptive measures
  - non-linearities reflect structure of the economy - can be affected by regulation
- Paper does not provide a complex answer, just another piece of empirical evidence.

# Literature

- t-distributed shocks:
  - DSGE models: Chib and Ramamurthy (2014), Cúrdia et al. (2014)
  - VARs: Chiu et al. (2014)
- Non-linearities:
  - DSGE models: financial accelerator (Bernanke et al., 1996)
  - Statistical models: McCallum (1991), Balke (2000), Hubrich and Tetlow (2014)
- Models with t-dist. shocks linear/linearized, the only non-linearity is represented by stochastic volatility of shocks.
- Non-linear models usually assume normal distribution of shocks.

# Contribution and results

- Accounting for t-distributed shocks, regime switch in shock propagation mechanism and regime switch in shock volatility (and addressing concerns related to 'small-scale' model).
- Results:
  - strong evidence of fat tails
  - fat tails more important than non-linearities in terms of model data fit
  - role for non-linearities in density forecasting

# Model

- Flexible enough to distinguish between regime switch in shock propagation mechanism, shock volatility and at the same time allow for *t*-distributed shocks.
- Simple enough to estimate all model parameters (important for density forecasting!) and avoid overfitting.

Threshold VAR:

$$
\begin{aligned}
y_t &= \sum_{i=1}^{R} x_{t,p_i} B_i I\left[r_{i-1} < y_{t-d}^{TR} < r_i\right] + u_t, \\
u_t &\sim MN\left(0, \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} < r_i\right] \Sigma_i\right) \\
\text{or } u_t &\sim MT\left(0, \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} < r_i\right] \Sigma_i, n\right)
\end{aligned}
$$

# Model

- Random variable distributed as multivariate $t$ distribution can be viewed as normally distributed with stochastic volatility:

$$
\begin{aligned}
\omega_t &\sim \Gamma\left(\frac{n}{2}, \frac{2}{n}\right) \\
u_t &\sim MN\left(0, \omega_t^{-1} \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} < r_i\right] \Sigma_i\right)
\end{aligned}
$$

# Data and Estimation

- Data set: quarterly data 1984Q1-2013Q4 (1964Q1-2013Q4, 1984Q1-2008Q2)
- Output growth, inflation, federal funds rate, measure of credit/financial market conditions.
- Credit/financial market conditions: BAA spread (Mix variable, Financial conditions index - FCI). Robustness wrt this indicator important to address the concern about small-scale model.
- Estimation: Gibbs sampler (Chen and Lee, 1995) with Metropolis step (Koop and Potter, 2003) and adaptive rejection sampling (Gilks and Wild, 1992).
- Priors: independent Normal-inverse Wishart, Beta for thresholds, Gamma for degrees of freedom, multinomial for delay parameter.
- 100 000 iterations for inference, 50 000 burn-in period
- One regime or two regimes assumed.

- In-sample fit measured by the Deviance Information Criterion (DIC).
- Specifications with *t*-distributed shocks preferable.
- For normal shocks regime switch helps to explain data for Mix variable and FCI.
- So, imposing normality can suggest presence of non-linearity which is a consequence of ignored fat tails.

Table: DIC (quarterly data, 1984Q1-2013Q4)

| | BAA spread | | Mix variable | | FCI | |
|---|---|---|---|---|---|---|
| | Number of regimes | | Number of regimes | | Number of regimes | |
| Shocks: | 1 | 2 | 1 | 2 | 1 | 2 |
| Normal | 666.26 | 1258.36 | −154.30 | −190.61 | 744.37 | 701.13 |
| t-dist. | 476.23 | 519.17 | −337.05 | −288.22 | 530.95 | 639.90 |

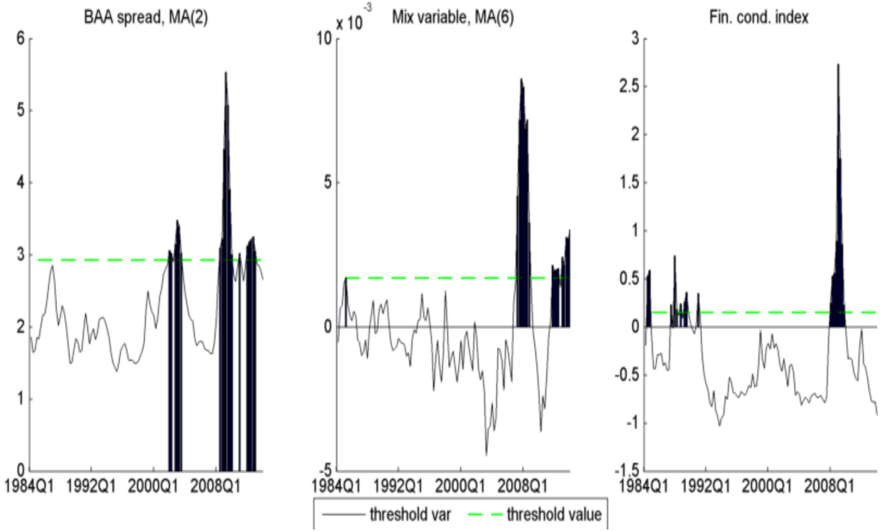
# Additional results

## In-sample fit

- Excluding the Great Recession provides similar results (lower need for fat tails).
- Extending data set (start by 1964Q1) provides similar results.
- Strong need for fat tails ($n$ close to 5), allowing for regime change in shock volatility does not lower fat-tailedness.
- Quarterly vs. monthly data (Stock and Watson, 2012, vs. Sims, 2012): still minor role of non-linearities if monthly data used for estimation.

# Results

## In-sample fit



- Three regimes (specification with quarterly data and BAA spread).
- Table suggests that models with more regimes (TVP-VAR) are perhaps not necessary:

Table: DIC

| | Number of regimes | | |
|---|---|---|---|
| Shocks: | 1 | 2 | 3 |
| Normal | 666.26 | 1258.36 | 1319.29 |
| t-dist. | 476.23 | 519.17 | 597.00 |

- Pseudo out-of-sample point/density forecasting exercise based on windows ending at 2002Q4 - 2013Q4.
- One-period-ahead forecasts compared with 45 ex-post observations, two-period-ahead forecasts with 44 observations, etc.
- Point/density forecasts simulated within the run of the Gibbs sampler - iterated forecasts for up to 7 quarters.
- Measure of point forecasting accuracy: root mean square error.
- As a measure of accuracy of density forecasts is used Kullback-Leibler Information Criterion.
- So, looking for model that yields the highest average logarithmic score $\frac{1}{N}\sum \ln f_{t+h,t}\left(\overline{x}_{t+h}\right)$.

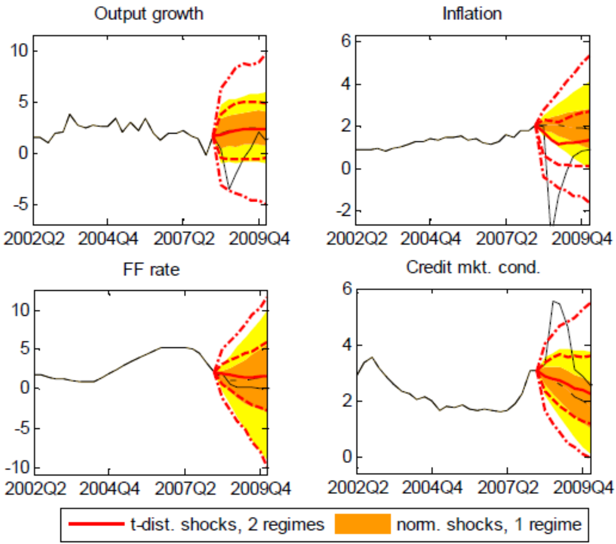
# Results

## Out–of-sample fit

- Basically forecasting accuracy measured for 4 variables and 7 forecasting horizons (28 cases).
- Density forecasts: for majority of cases the specification with regime switch and/or t-distributed shocks are preferred.
- When focusing on point forecasts, linear model with normally distributed errors preferred approximately in half of the cases.
- **So, non-linearities and fat tails improve tails of density forecasts (i.e. we get more accurate forecasts of extreme events).**
- Results are not driven by the Great Recession (evaluation on sub-sample 2002Q4-2008Q2 results in shift of preference towards non-linear models and t-distributed shock distributions).

# Results

## Out–of-sample fit



# Conclusions

- First attempt to account for all possible reasons of extreme events.
- Econometric/computational reasons force us to use small-scale model with a simple way how to account for non-linearities.
- Non-linearities and fat-tails in error distributions lead to more accurate tails of density forecasts.
- Application: probabilistic evaluation of macro scenarios in stress testing.